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square is placed above and to the right of the two circles, respectively. Also let θ be the angle which the side tangent to the former circle makes with the x-axis. The limiting positions of the square will be when the heel is just in contact with one of the circular pegs; that is, for example, when $a \cos \theta = r + a \sin \theta$, or $\theta = \frac{\pi}{4} - \sin^{-1} \frac{r}{\sqrt{2}a}$, so that the physical limits of θ are $\pm \left(\frac{\pi}{4} - \sin^{-1} \frac{r}{\sqrt{2}a} \right)$.

The points of tangency for any position of the square will be $(a - r \sin \theta,\ r \cos \theta)$ and $(r \cos \theta,\ a + r \sin \theta)$ and the equations of these tangents will be, respectively,

$$-(x - a) \sin \theta + y \cos \theta = r \qquad \text{and} \qquad x \cos \theta + (y - a) \sin \theta = r,$$

from which we get

$$x = - r(\sin \theta - \cos \theta) + a \sin \theta(\sin \theta + \cos \theta),$$
$$y = r(\sin \theta + \cos \theta) + a \sin \theta(\sin \theta - \cos \theta)$$

for the parametric equations defining the curve, with $-\left(\frac{\pi}{4} - \sin^{-1} \frac{r}{\sqrt{2}a}\right) \leqq \theta \leqq \left(\frac{\pi}{4} - \sin^{-1} \frac{r}{\sqrt{2}a}\right)$ for that portion subject to the physical limitations of the problem. By elimination of θ, we get for the general equation of the curve[1]

$$2r^2[x^2 + y^2 - 2a(x + y) + 2a^2] = [x^2 + y^2 - ax - ay]^2.$$

If in this $r = 0$, we have the circle $x^2 + y^2 = ax + ay$, as we should. For the above limits of θ to be correct, the shorter arm (18 inches) of the square must be $> \sqrt{(2a^2 - r^2)} - r$.

2895 [1921, 184]. Proposed by R. M. MATHEWS, Wesleyan University.

To construct an equilateral triangle with its vertices lying on: (*a*) any three coplanar lines; (*b*) three parallels in space; and (*c*) any three lines in space.

SOLUTION BY F. L. WILMER, Omaha, Neb.

(*a*) Let the three lines be l_1, l_2 and l_3. When they are not parallel we may suppose that the first two intersect in O. We bisect one of the angles between them and let the bisector meet l_3 in C. Then draw two lines from C, making angles of 30 degrees with CO (or with OC produced) and cutting l_1 and l_2 in A and B, respectively. ABC will be an equilateral triangle and there will generally be four triangles that can be constructed in this way.

When the three lines are parallel, draw a secant cutting them in P_1, P_2, P_3, respectively, and construct an equilateral triangle with P_1P_3 as one side and Q, say, as the third vertex. Rotate this triangle about P_2 until Q falls on l_2 and let the triangle in the new position be $P_1'Q'P_3'$. If we denote by A and B the points where $P_1'P_3'$ cuts l_1 and l_3, respectively, we shall find that the equilateral triangle ABC, constructed on AB on the same side of AB as Q', will satisfy the conditions of the problem; for $P_2P_1'/P_2A = P_2P_3'/P_2B$. Therefore P_2 is a center of similitude for the triangles $P_1'Q'P_3'$ and ABC, and C lies on P_2Q' or l_2.

(*b*) Consider the prismatic surface with edges l_1, l_2, l_3 and suppose that an equilateral triangle of side L can be placed with its three vertices on these three lines, respectively. Let a, b, c be the lengths of the sides of a right section $a \leqq b \leqq c$. Then

$$\sqrt{L^2 - a^2} = \sqrt{L^2 - b^2} + \sqrt{L^2 - c^2} \tag{1}$$

or

$$3L^4 - 2(a^2 + b^2 + c^2)L^2 + 16A^2 = 0, \tag{2}$$

where A is the area of the right section.

If L^2 is replaced by c^2, the expression in the first member of (2) reduces to $-(b^2 - a^2)^2$ and this shows that the equation in L^2 has two positive roots, one less than c^2 and the other greater than c^2 (unless $a = b$). The root greater than c^2 makes the three terms of (1) all real and satisfies (1) as written, since the first term of (1) is greater than or equal to each of the others.

(*c*) Partial solution—Instead of the three given lines, we may consider two non-coplanar lines and a point. Take the common perpendicular of the two for the z-axis and the bisectors of the angles made by their projections on the mid-plane as the axes of x and y. Then we may let

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[1] If we take the line joining the centers of the circles as the x-axis, with the origin at the midpoint of that line, the equation will be $(x^2 + y^2 - b^2)^2 = 2r^2[x^2 + (y - b)^2]$, where $2b$ is the distance between the centers.—EDITORS.

(x_1, px_1, h), $(x_2, -px_2, -h)$ and (α, β, γ) be the vertices of an equilateral triangle of side L and there will be three expressions for L^2 which give the three equations

$$\begin{aligned}(1 + p^2)(x_1^2 + x_2^2) - 2(1 - p^2)x_1x_2 &= L^2 - 4h^2,\\ (1 + p^2)x_1^2 - 2(\alpha + p\beta)x_1 &= L^2 - \alpha^2 - \beta^2 - (\gamma - h)^2,\\ (1 + p^2)x_2^2 - 2(\alpha - p\beta)x_2 &= L^2 - \alpha^2 - \beta^2 - (\gamma + h)^2.\end{aligned}$$

Eliminating x_1 and x_2 from these three equations, we have an equation of the fourth degree in L^2 with coefficients which are functions of α, β, γ.

NOTE BY THE EDITORS.—There are an infinite number of solutions in (*a*) when the lines are not parallel, as well as when they are parallel. This may be shown as follows: draw an equilateral triangle ABC with A on l_1 and B on l_2. With a given position of AB, the vertex C may lie on either side of AB. One at least of the straight lines OC will cut l_3 in the point C', unless these are both parallel to l_3 and so the same line, and this can be avoided by shifting the position of the triangle. The lines $C'A'$ and $C'B'$, drawn from C' to l_1 and l_2, parallel respectively to CA and CB, will be two sides of the triangle $A'B'C'$ which is equilateral and satisfies the conditions of the problem. The triangles $A'B'C'$ for different triangles ABC will vary in size, for if they were all equal the locus of C' would be an ellipse and not the line l_2.

In (*c*) the construction is not always possible. If, for example, the given point is at the origin, $\alpha = \beta = \gamma = 0$, then $x_2 = \pm x_1$, and $L^2 = 4h^2/(1 - 3p^2)$ or $4p^2h^2/(p^2 - 3)$. It is necessary in this case, therefore, that the acute angles between the projections of the two lines shall be less than 60 degrees.

NOTES AND NEWS.

It is hoped that readers of the MONTHLY will coöperate in contributing to the general interest of this department by sending items to H. P. MANNING, 69 Weymouth St., Providence, R. I.

Miss MAY J. SPERRY, of Knox College, has been appointed instructor at the University of Syracuse.

Miss MARGARET C. PACKER, of Brown University, has been appointed instructor at Hood College, Fredericksburg, Md.

Miss FRANCES M. MERRIAM, of Brown University, has been appointed instructor of mathematics at Wellesley College.

Assistant professor J. W. HARRELL, of Baylor University, Waco, Texas, is on leave of absence studying at Brown University.

MARCUS SKARSTEDT, of Augustana College, has been appointed professor of mathematics in Whittier College.

Miss MINNIE W. CALDWELL has been appointed head of the department of mathematics in Chowan College, Murfreesboro, N. C.

Associate Professor E. L. DODD will teach at Williams College during 1922–1923 on leave of absence from the University of Texas.

C. G. SIMPSON, of the Carnegie Institute of Technology, has been appointed professor of mathematics in the Milwaukee College of Electrical Engineering.

J. G. FOWLKES has been made assistant professor of education in the University of Wisconsin.

R. L. CHARLES, associate professor of physics at Lehigh University, has been appointed professor of physics and electricity at Franklin and Marshall College.

At Brown University, W. R. BURWELL, of the University of Tennessee, has been appointed assistant professor of mathematics and dean of freshmen; J. H. FITHIAN, of Taft School, Watertown, Conn., and H. C. HICKS, of the University of Chicago, have been appointed instructors.